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Abstract

In this paper, we propose a path relinking procedure for the fixed-charge capacitated multicommodity network design problem. Cycle-based neighbourhoods are used both to move along path between elite solutions and to generate the elite candidate set by a tabu-like local search procedure. Several variants of the method are implemented and compared. Extensive computational experiments indicate that the path relinking procedure offers excellent results. It systematically outperforms the cycle-based tabu search method in both solution quality and computational effort and offers the best current meta-heuristic for this difficult class of problems.

Keywords : path relinking, tabu search, fixed charge capacitated multicommodity network design, meta-heuristics, cycle-based neighbourhoods

Résumé

Dans cet article, nous proposons une procédure de "path relinking" pour le problème de synthèse de réseau multiproduits avec capacités. Des mouvements basés sur des cycles sont utilisés autant pour explorer les chemins entre les solutions élites, que pour générer l'ensemble des solutions élites candidates par une procédure de recherche locale avec tabous. Nous développons et comparons plusieurs variantes de la méthode. Les résultats expérimentaux indiquent que la procédure proposée donne d'excellents résultats. Elle offre systématiquement de meilleures performances, en termes de qualité de solution et de temps de résolution que la méthode de recherche avec tabous basée sur des cycles, et offre la meilleure heuristique actuellement disponible pour cette classe de problèmes.

Mots-clés : path relinking, méthodes de recherche avec tabous, synthèse de réseau multiproduits avec capacités, méta-heuristiques, voisinage par cycles

1 Introduction

The fixed-charge capacitated multicommodity network design formulation (CMND) represents a generic model for a wide range of applications in planning the construction, development, improvement, and operations of transportation, logistics, telecommunication, and production systems, as well as in many other major areas (Balakrishnan, Magnanti, and Mirchandani 1997, Magnanti and Wong 1986, Minoux 1986). The problem is usually modeled as a combinatorial optimization problem and is NP-hard in the strong sense. Thus, not only the generation of optimal solutions to large problem instances constitutes a significant challenge, but even identifying efficiently good feasible solutions has proved a formidable task not entirely mastered.

A few authors have developed exact solution methods for CMND problems (e.g., Crainic, Frangioni, and Gendron 2001, Gendron, Crainic, and Frangioni 1998, Holmberg and Yuan 2000) but only heuristics have proved effective on reasonably large problems with many commodities. Crainic, Gendreau, and Farvolden (2000) proposed a tabu search meta-heuristic for the path-based formulation of the problem. The method explores the space of the path-flow variables by using pivot-like moves and column generation. Even though the method produced very impressive results, its search efficiency may be limited by the fact that each move considers the impact of changing the flow of one commodity only (a pivot from one path to another). Moreover, one cannot guarantee that all paths will be considered, neither explicitly, nor implicitly, which impairs the capabilities of the algorithm to explore thoroughly the search space.

Ghamlouche, Crainic, and Gendreau (2002) proposed a new class of neighbourhood structures for the CMND that overcomes some of these limitations. To evaluate these neighbourhoods, the authors implemented them into a very simple tabu-based local search procedure that currently appears as the best approximate solution method for the CMND in terms of robust performance, solution quality, and computing efficiency. This approach is not entirely satisfactory, however, since it does not go beyond a rather local exploration of the search space. In fact, Ghamlouche, Crainic, and Gendreau (2002) clearly state that their goal was to explore the behaviour of the neighbourhood class only, and that more refined search methods would have to be developed. The present paper is a contribution toward reaching this goal.

Path relinking has been proposed as a method to better explore the solution space of complex problems when a set of promising, or *elite* solutions is known (Glover 1997, Glover and Laguna 1997, Glover, Laguna, and Marti 2000). The fundamental idea of the method is to build a solution path between two elite solutions, using their attributes to guide the generation of intermediary solutions. This process is supposed to yield solutions that combine the "good" characteristics of the starting solutions and that improve the set of elite solution and, eventually, the overall best. The body of research reported on applications of path relinking is somewhat limited but appears to sustain these expectations.

The motivation of the research presented in this paper is to explore the adaptation of path relinking to the CMND. We aim, in particular, to evaluate the benefits of combining the path relinking framework and the cycle-based neighbourhood structures of Ghamlouche, Crainic, and Gendreau (2002) into a better meta-heuristic for the CMND. Numerical experiments support this claim and show that combining cycle-based tabu search and path relinking results in an enhanced and more effective search strategy.

To summarize, the contributions of this work are: 1) a new meta-heuristic for the CMND that outperforms the current best approximate methods; 2) a successful development of a path relinking method to a difficult combinatorial optimization problem; 3) an enhancement of our understanding of the behaviour of path relinking as a search method.

The outline of the paper is as follows. Section 2 starts with a formal definition of the problem and summarizes the relevant details of the cycle-based neighbourhoods and initial tabu search method. Following a brief outline of the path relinking methodology, Section 3 details our implementation for the CMND. Section 4 is dedicated to experimental results. We compare, in particular, the path relinking results to those obtained by the cycle-based tabu search to identify the effects of the new methodology on the quality of the solutions obtained and total computational time. We conclude in Section 5.

2 Background and Neighbourhood Overview

The goal of a CMND formulation is to find the optimal configuration - the links to include in the final design - of a network of limited capacity to satisfy the demand of transportation of different commodities sharing the network. (To simplify the presentation, we assume that design decisions concern only network links.) The objective is to minimize the total system cost, computed as the sum of the link fixed and routing costs.

The CMND can be formulated as follows. Let $\mathcal{G} = (\mathcal{N}, \mathcal{A})$ be a network with set of nodes $\mathcal{N}$ and set of directed arcs $\mathcal{A}$. Without loss of generality, we assume that all $(i, j) \in \mathcal{A}$ are design arcs. Let $\mathcal{P}$ denote the set of commodities to move using this network, where each commodity p has a single origin $o(p)$, a single destination $s(p)$, and a flow requirement of w^p units between its origin and destination nodes. The arc-based formulation of the CMND can then be written as

$$\min z(x, y) = \sum_{(i,j)\in\mathcal{A}} f_{ij} y_{ij} + \sum_{p\in\mathcal{P}} \sum_{(i,j)\in\mathcal{A}} c_{ij}^p x_{ij}^p \tag{1}$$

$$\text{subject to} \sum_{j\in\mathcal{N}^+(i)} x_{ij}^p - \sum_{j\in\mathcal{N}^-(i)} x_{ji}^p = d_i^p \quad \forall i \in \mathcal{N}, \forall\, p \in \mathcal{P}, \tag{2}$$

$$\sum_{p\in\mathcal{P}} x_{ij}^p \leq u_{ij} y_{ij} \quad \forall (i,j) \in \mathcal{A}, \tag{3}$$

$$x_{ij}^p \geq 0 \quad \forall (i,j) \in \mathcal{A}, \forall p \in \mathcal{P}, \tag{4}$$

$$y_{ij} \in \{0,1\} \quad \forall (i,j) \in \mathcal{A} \tag{5}$$

where y_{ij}, $(i,j) \in \mathcal{A}$, represent the design variables that equal 1 if arc (i,j) is selected in the final design (and 0 otherwise), x_{ij}^p stand for the flow distribution decision variables indicating the amount of flow of commodity $p \in \mathcal{P}$ on arc (i,j), and

$\mathcal{N}^+(i)/\mathcal{N}^-(i)$: Set of outward/inward neighbours of node i,
f_{ij},u_{ij} : Fixed cost and capacity applied on arc (i,j),
c_{ij}^p : Cost of one unit flow of commodity p on arc (i,j),

$$d_i^p = \begin{cases} w^p & if \quad i = o(p) \\ -w^p & if \quad i = s(p) \\ 0 & \text{otherwise.} \end{cases}$$

The objective function (1) accounts for the total system cost, the fixed cost of arcs included in a given design plus the cost of routing the product demand, and aims to select the minimum cost design. Constraints (2) represent the network flow conservation relations, while constraints (3) state that for each arc, the total flow of all commodities cannot exceed its capacity if the arc is opened ($y_{ij} = 1$) and must be 0 if the arc is closed ($y_{ij} = 0$). Relations (5) and (4) are the usual non-negativity and integrality constraints for decision variables.

Recall that, for a given design vector $\bar{y}$, the arc-based formulation of the CMND becomes a capacitated multicommodity minimum cost flow problem (CMCF)

$$\min\, z(x(\bar{y})) = \sum_{p\in\mathcal{P}} \sum_{(i,j)\in\mathcal{A}(\bar{y})} c_{ij}^p x_{ij}^p \tag{6}$$

subject to (2) plus

$$\sum_{p\in\mathcal{P}} x_{ij}^p \leq u_{ij}\bar{y}_{ij} \quad \forall (i,j) \in \mathcal{A}(\bar{y}),$$

$$x_{ij}^p \geq 0 \quad \forall (i,j) \in \mathcal{A}(\bar{y}), \forall p \in \mathcal{P},$$

where $\mathcal{A}(\bar{y})$ stands for the set of arcs corresponding to the design $\bar{y}$. A solution to the CMND may then be viewed as an assignment $\bar{y}$ of 0 or 1 to each design variable, plus the optimal flow of the corresponding multicommodity minimum cost flow problem $x^*(\bar{y})$. Similarly, the objective function value associated to a solution $(\bar{y}, x^*(\bar{y}))$ is the sum of the fixed cost of the open arcs in $\bar{y}$ and the objective function value of the CMCF associated to $x^*(\bar{y})$

$$z(\bar{y}, x^*(\bar{y})) = \sum_{(i,j)\in\mathcal{A}(\bar{y})} f_{ij}\bar{y}_{ij} + z(x^*(\bar{y})). \tag{7}$$

The cycle-based neighbourhood structures for the CMND proposed by Ghamlouche, Crainic, and Gendreau (2002) explore the space of the arc design variables by re-directing flow around cycles and closing and opening design arcs accordingly. The basic neighbourhood defines moves that may explicitly take into account the impact on the total design cost of potential modifications to the flow distribution of several commodities simultaneously. The fundamental idea is that one may move from one solution to another by 1) Identifying two points in the network together with two paths connecting these points, thus closing a cycle; 2) Deviating the total flow from one path to another such that at least one currently open arc becomes empty; 3) Closing all previously open arcs in the cycle that are empty following the flow deviation and, symmetrically, opening all previously closed arcs that now have flow. Such neighbourhoods are huge, however, and their explicit and exhaustive exploration is not practical in most situations. Moreover, the complete evaluation of any design modification involves the resolution of a capacitated multicommodity network flow problem, which rapidly becomes extremely computation intensive. Thus, in order to select the best move for a given solution, the method implements an efficient procedure that avoids the complete evaluation of every examined move and generates a limited number of cycles that include the "good" moves.

Indeed, not all cycles are of equal interest. The method seeks, in particular, moves that modify the status of several arcs and that lead to a significant modification of the flow distribution. Therefore, moves that close at least one arc and open new paths for a group of commodities appear attractive. To close an arc (i, j), one must be able to deviate all its flow. Let the *residual capacity* of a cycle denote the maximum flow one can deviate around the cycle. The residual capacity of any cycle that includes (i, j) must then be at least equal to $\sum_{p \in \mathcal{P}} x_{ij}^p$, the total flow on arc (i, j). Consequently, the cycles of interest are those that display a residual capacity equal to one of the values in Γ defined as the set of the total (strictly positive) volumes on the open arcs of the corresponding network:

$$\Gamma(\bar{y}) = \left\{ \sum_{p \in \mathcal{P}} x_{ij}^p > 0 \ : \ (i, j) \in \mathcal{A}(\bar{y}) \right\}. \tag{8}$$

Cycles are thus identified on *residual networks* of $\mathcal{A}(\bar{y})$ defined according to $\Gamma(\bar{y})$. The best move in the neighbourhood of $\bar{y}$ is then identified by the cycle leading to the network modification that yields the largest improvement (smallest deterioration, eventually) in the objective function (1). To reduce the computational burden, cycles are identified and evaluated for a set of candidate links. The "lowest" cost cycle for each candidate link is identified by an optimization heuristic based on a modification of the shortest path label-correcting algorithm that avoids getting trapped in negative directed cycles. The method thus progressively builds a set of good candidate neighbours (cycles) among which the best move is selected.

Let $\mathcal{C} \subseteq \mathcal{A}$ denote the set of *candidate* links, where each arc $(i, j) \in \mathcal{C}$ will be considered as the starting point of cycles. Let $\mathcal{C}(\gamma) \subseteq \mathcal{C}$ include all arcs $(i, j) \in \mathcal{C}$ that can support a movement of γ units of flow. A closed arc may belong to $\mathcal{C}(\gamma)$ only if its capacity is at least

1: Build set $\Gamma(\bar{y})$

2: For each $\gamma \in \Gamma(\bar{y})$

    Build the γ-residual network

    For each arc $(i, j) \in \mathcal{C}(\gamma)$

        find the lowest cost cycle using a network optimization procedure

3: Select and implement the best move

4: Evaluate new solution

Figure 1: Neighbourhood Exploration

γ. For an open arc, either its flow or its residual capacity must be at least γ units. The heuristic procedure proposed to explore the neighbourhoods of a given solution $\bar{y}$, given a set of candidate links $\mathcal{C}(\bar{y})$, is synthesized in Figure 1.

To evaluate these concepts, Ghamlouche, Crainic and Gendreau (2001) developed a simple tabu search-based local search procedure that integrates two versions of the cycle-based neighbourhood. The basic one corresponds to the definition above and considers the flow of all commodities when determining cycles. The second one refines the search by implementing moves that result from the deviation of the flow of one commodity at a time (the set Γ is instantiated for each commodity, denoted Γ^p, to contain the flow of the particular commodity that exists on each arc of the network), and is used to intensify the search.

Following an initialization phase, the procedure explores the search space defined by the design variables using a simple local search framework with tabu tags. At each iteration, the best non-tabu move is determined and implemented, whether it improves the overall solution or not. A short-term tabu memory is used to record characteristics of visited solutions to avoid cycling. When a particularly good solution is encountered, the search is intensified using the particular neighbourhood implementation indicated above, which considers the flow distribution of one commodity only. A solution is considered particularly good when it either improves the best overall solution or is close to it by at least a pre-defined percentage. The method terminates whenever a predefined stopping criterion (number of iterations, CPU time, etc.) is met. Computational results on a large set of instances with various characteristics, show that the cycle-based tabu search is powerful. It is in fact the best current heuristic for the CMND.

The cycle-based tabu search has been extensively calibrated and tested. This experimental study yielded the parameter settings that are also used to implement the path relinking presented in the following section.

3 Path Relinking

Introduced by Glover (1997; see also Glover and Laguna 1997 and Glover, Laguna, and Marti 2000), *path relinking* is a meta-heuristic that operates on a set of elite solutions, called the *reference set*, and generates paths between solutions in this set to create improved new ones. Starting from an *initial* solution, the primary goal of the search is to find a path towards another, *guiding* solution, by performing moves that progressively introduce into the current solution attributes of the guiding solution. Thus, the method does not progress by choosing a "best" move from the neighbourhood, but rather by selecting the "best" move from a restricted set of moves that incorporate some or all of the "good" attributes of the guiding solution. This exploration allows the search to perform moves that may be considered unattractive according to the objective function value but which appear essential in reaching solutions with given characteristics.

Path relinking has been proposed in connection with tabu search. Very little computational investigation of this method has been done, yet. Laguna and Marti (1999) apply path relinking within a GRASP framework to the problem of minimizing the number of arc crossings in a bipartite graph. Results are very encouraging: The authors report that path relinking improved the performance of the GRASP implementation. Laguna, Marti and Campos (1999) propose an algorithm based on tabu search with strategies for search intensification and diversification for the linear ordering problem. The method uses path relinking for search intensification. Elite solutions are selected as the best solutions found during the entire search, while initial solutions are local optima. The authors report that combining tabu search and path relinking is beneficial. They also indicate, however, that one must find the "right" balance between search intensification and diversification in order to obtain the best results. Ribeiro, Uchoa and Werneck (2000) propose a hybrid GRASP with perturbations and path relinking for the steiner problem in graphs. Their results indicate that path relinking is effective and systematically leads to improvements in the solutions found at the end of the hybrid GRASP perturbation.

This paper investigate the implementation of path relinking to the CMND. In our implementation, the method starts with a tabu search phase using the algorithm of Ghamlouche, Crainic, and Gendreau (2001) to build the reference set. When a predefined number of consecutive moves without improvement is observed, the method switches to a path relinking phase that stops when the reference set becomes empty (cardinality ≤ 1). Then, either stopping conditions are verified, or the procedure is repeated to build a new reference set. The procedure is schematically shown in Figure 2.

The path relinking method is made up of the following three main components:

- Identification of the reference set.
- Choice of the initial and guiding solutions.
- Identification of the neighbourhood structure and the guiding attributes.

1: Generate a starting set of solutions using the cycle-based tabu search

2: Designate a subset of solutions to be included in the reference set

While the reference set cardinality > 1

- Select two solutions from the reference set
- Identify initial and guiding solutions
- Remove initial solution from the reference set
- Move from initial toward guiding solution, generating intermediate solutions
- Update the reference set

3: Verify stopping criterion: Stop or go to 1:

Figure 2: General Path Relinking Procedure

The following sub-sections detail our implementation of these components to address the CMND. Section 3.4 sums up the overall path relinking procedure.

3.1 Identification of the Reference Set

What solutions are included in the reference set $\mathcal{R}$, how good and how diversified they are, have a major impact on the quality of the new generated solutions. In our implementation, $\mathcal{R}$ is built during the tabu search phase and subsequently enriched during the path relinking phase. We study six strategies corresponding to different ways to build $\mathcal{R}$.

In strategy *S1*, $\mathcal{R}$ is built using each solution that, at some stage of the tabu search phase, improves the best overall solution and become the best one. The goal of this strategy is to link *overall improving* solutions. Solutions 1, 2, 3, and 4 of Figure 3 are selected by this strategy.

Strategy *S2* retains the *"best" local minima* found during the tabu search phase. This strategy is motivated by the idea that local minimum solutions share characteristics with optimum solutions. One then hopes that linking such solutions yields improved new ones. In Figure 3, solutions 2, 3, 4, and 5 are members of $\mathcal{R}$ when its dimension is limited to 4.

Strategy *S3* selects $\mathcal{R}$*-improving* local minima, that is local minimum solutions that offer a better evaluation of the objective function than those already in $\mathcal{R}$. The idea is to introduce a time dimension in the selection process - often better solutions are encountered once the search has proceeded for some time - without loosing potential good solutions found early on during the search. In Figure 3, if solutions 1 and 2 are already in $\mathcal{R}$, only solutions 3 and 4 are admitted to this set.

Strategy *S4* represents a first attempt to allow solutions to be retained in $\mathcal{R}$ not only

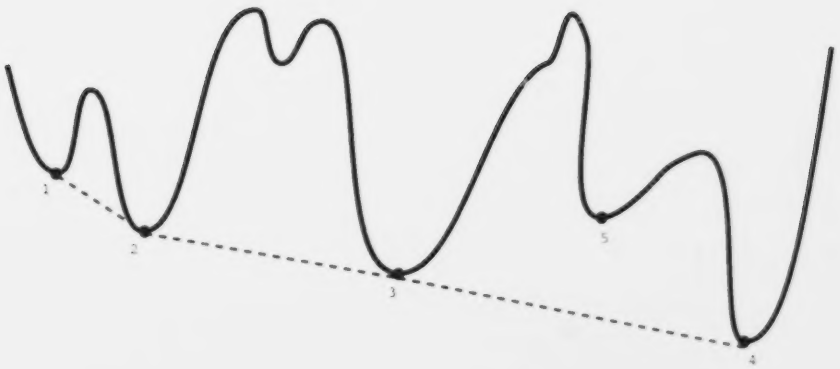


Figure 3: Building the Reference Set Example

according to an attractive solution value but also according to a *diversity*, or *dissimilarity* criterion. Define D_s^b, the level of dissimilarity between solution s and the best solution b, as the number of arcs with different status between the two solutions:

$$D_s^b = \sum_{(i,j)\in A} d_{ij},$$

where

$$d_{ij} = \begin{cases} 0 & if \quad y_{ij}^s = y_{ij}^b \\ 1 & \text{otherwise.} \end{cases}$$

Define also the median position of all solutions $s_i \in \mathcal{R}$ relatively to the best solution b as:

$$Median = \frac{\sum_{s_i \in \mathcal{R}}^{s_i \neq b} D_{s_i}^b}{|\mathcal{R}| - 1},$$

where $|\mathcal{R}|$ denotes the dimension of the reference set. A solution s is then included in $\mathcal{R}$ if it improves the best overall solution, or if it improves the worst solution and its level of dissimilarity exceeds the median, $D_s^b > Median$. The procedure of Figure 4 is used to implement this strategy.

Strategy *S5* aims to ensure both the *quality* and the *diversity* of solutions in $\mathcal{R}$. This strategy is pointed out by Laguna and Armentano (2001) as the most important lesson they learned related to the way the reference set should be initialized. Starting with a large set of ("good") solutions S, $\mathcal{R}$ is partially filled with the best solutions found, to satisfy the purpose of quality. It is then extended with solutions that change significantly the structure of the solutions already in $\mathcal{R}$ to ensure diversity. Strategy *S5* thus builds the reference set according to the following steps:

1. Initialize the reference set $\mathcal{R}$ with solutions satisfying strategy *S1*.

For a given candidate solution s

If $\mathcal{R}$ is not full, Add s to $\mathcal{R}$

Else, If s is the best overall solution,
Replace the worst solution in $\mathcal{R}$ by s

Else, If s is better than the worst solution in $\mathcal{R}$

- Compute $Median$
- Compute D_s^b
- If $D_s^b > Median$,
  Replace the worst solution in $\mathcal{R}$ by s

Figure 4: Building $\mathcal{R}$ According to Strategy *S4*

2. For each solution $s \in \{S \backslash \mathcal{R}\}$, compute the level of diversity Δ_s^R between solution s and all solutions $s_i \in \mathcal{R}$: $\Delta_s^R = \sum_{s_i \in \mathcal{R}} D_{s_i}^s / \mathcal{R}$

3. Extend $\mathcal{R}$ with solutions $s \in \{S \backslash \mathcal{R}\}$ that maximize Δ_s^R.

Strategy *S6* proceeds similarly to *S5* with the difference that $\mathcal{R}$ is extended with solutions *close* to those already in $\mathcal{R}$. This strategy aims to *intensify* the search by grouping good solutions with similar designs To build $\mathcal{R}$, strategy *S6* implements the first two steps used for strategy *S5*, then extends it with solutions $s \in \{S \backslash \mathcal{R}\}$ that minimize Δ_s^R.

3.2 Choice of the Initial and Guiding Solutions

The choice of the initial and guiding solutions is also critical to the quality of the new solutions and, thus, the performance of the procedure. We investigate the effect of the following criteria:

C1: Guiding and initial solutions are defined as the *best* and *worst* solutions, respectively.

C2: Guiding solution is defined as the *best* solution in the reference set, while the initial solution is the *second best* one.

C3: Guiding solution is defined as the *best* solution in the reference set, while the initial solution is defined as the solution with *maximum Hamming distance* from the guiding solution.

C4: Guiding and initial solutions are chosen *randomly* from the reference set.

C5: Guiding and initial solutions are chosen as the *most distant* solutions in the reference set.

C6: Guiding and initial solutions are defined respectively as the *worst* and the *best* solutions in the reference set.

3.3 Neighbourhood Structure and Guiding Attributes

Recall (Section 2) the two versions of the cycle-based neighbourhood used in the tabu search procedure: the basic one that considers all flows on arcs, while the intensification version deviates only a commodity at the time. The former brings about significant modifications to the design structure, while the latter yields more limited changes. The aim of the path relinking procedure is to introduce progressively attributes of the guiding solution into new solutions obtained by moving away from an initial point. This requires small but purposeful movements and, thus, we use the second variant of the neighbourhood structure considering flow deviations of only one commodity at a time. Consequently, for the path relinking phase, the set Γ becomes Γ^p, defined as the set of the positive flow of commodity p on each of the open arcs of the corresponding network:

$$\Gamma^p(\bar{y}) = \left\{ x_{ij}^p > 0 \ : \ (i,j) \in \mathcal{A}(\bar{y}),\ p \in \mathcal{P} \right\}. \tag{9}$$

Moves from the current solution to a neighbouring one during path relinking must direct the search towards the guiding solution. Thus, cycles of interest are those that introduce attributes (i.e, arcs) present in the guiding solution. To apply this aggressive orientation judiciously, the set of the candidate links $\mathcal{C}$ becomes $\mathcal{C}_r$, the set of all arcs with a different status in the initial solution s_i and the guiding solution s_g:

$$\mathcal{C}_r = \{(i,j) \in \mathcal{A} : y_{ij}^{s_i} \neq y_{ij}^{s_g}\}. \tag{10}$$

Similarly, sets $\mathcal{C}_r(\gamma) \subseteq \mathcal{C}_r$ include now all arcs $(i,j) \in \mathcal{C}_r$ that can support a movement of γ units of flow. To select among the various possible paths between the two reference solutions, we move from the initial solution toward the guiding solution by selecting the "best" move at each step. That is, cycles that include arcs in $\mathcal{C}_r$ are evaluated and a move is performed toward the "best" one. This neighbour becomes the new current solution, even if it is worst than the previous one.

3.4 The Overall Procedure

To sum up, the procedure combines cycle-based tabu search and path relinking and alternates between the two. After an initialization phase, the search proceeds with the tabu search to

Initialization

Generate an initial feasible solution to initiate *BestSolution* and *CurrentSolution*

Main search loop

Repeat the following until a stopping condition is met

- Perform one tabu search iteration to get a new current solution
- Add current solution to $\mathcal{R}$ if it satisfies the requirements of strategy S_i
- If path relinking is to be launched,
  - Extend $\mathcal{R}$ if strategy *S5* or *S6* is used
  - Perform *Main Path Relinking Loop* as shown in Figure 6

Figure 5: Path Relinking Procedure

initialize the reference set $\mathcal{R}$ according to one of the strategies of Section 3.1. The method then switches to a path relinking phase. Initial and guiding solutions are selected in $\mathcal{R}$ according to one of the strategies of Section 3.2. The search proceeds by moving towards the guiding solution as indicated in Section 3.3 and Figure 6.

To select the "best" move in the neighbourhood of the current solution $\bar{y}$ during the path relinking phase, the procedure first determines the set of flow deviations $\Gamma^p(\bar{y})$ and the set of candidate links C_r as defined in (9) and (10) respectively. Then, for each commodity $p \in P$, a γ-residual network is built for each flow value $\gamma \in \Gamma^p$ and a low cost cycle is found for each arc $(i,j) \in \mathcal{C}_r(\gamma)$. If the lowest-cost cycle for commodity p is negative, the procedure updates the current solution, assigns a tabu status to each complemented arc, and examines the next product. If no negative cycle is found for any commodity, the procedure selects the cycle with lowest-cost overall cost and implements it. A new relinking path is initiated when no cycle is found between the current and guiding solutions.

Notice that, due to the nature of the neighbourhoods used, there is no guarantee that the guiding solution will be reached. One cannot, therefore, stop the process only if the current and the guiding solutions are the same. We then define Δ_{IG} as the number of arcs with different status between the initial and the guiding solutions and we allow the search to explore a number of solutions not larger than Δ_{IG}.

Computing the solution of the capacitated multicommodity network flow sub-problem constitutes a significant computational effort. On the other hand, since modifying the flow of one commodity around a cycle does not, in most cases, induce large variations in the design of the network, the CMCF is recomputed only when no more negative-cost cycles are found. To further reduce the computational burden, the reference set $\mathcal{R}$ is updated only with the best solutions found during a path relinking exploration, and only if these solutions improve the best overall solution (always included in $\mathcal{R}$).

1: While the reference set cardinality > 1

- Identify *InitialSolution* and *GuidingSolution* according to a strategy C_i
- Set *CurrentSolution* to *InitialSolution* and remove the latter from $\mathcal{R}$
- Compute Δ_{IG}

2: While ($\Delta_{IG} \neq 0$ AND *CurrentSolution* $\neq$ *GuidingSolution*)

- Set *BestPositiveCycle* $= \infty$
- Repeat until no negative cycle is detected in any γ-residual network
  - ▶ For each commodity $p \in P$ and flow value $\gamma \in \Gamma^p$
    - Build the γ-residual network
    - Find a low cost cycle for each arc $(i, j) \in \mathcal{C}_r(\gamma)$, $\gamma \in \Gamma^p$
    - Select the best overall cycle, γ_{best} the associated volume of deviated flow
    - If the selected cycle is improving (negative cost) update the solution by:
      - Modify the flow around the selected cycle by a quantity γ_{best};
      - Open/close appropriate arcs;
      - Update the cost of *CurrentSolution* by adding the cost of the cycle (without solving the CMCF).
      - Assign a tabu status to each complemented arc.
    - Otherwise, update *BestPositiveCycle* (positive cost cycle)
- If no improving cycle was found
  - ▶ If *BestPositiveCycle* $< \infty$, implement corresponding move
    - Open/close appropriate arcs
    - Assign a tabu status to each complemented arc
  - ▶ Otherwise, go to 1 (null cycle found)
- Solve exactly the CMCF associated to the current design to obtain the new *CurrentSolution*
- $\Delta_{IG} = \Delta_{IG} - 1$
- Update *BestSolution* (i.e., cost(*CurrentSolution*) $<$ cost(*BestSolution*))
- Update $\mathcal{R}$ if cost(*BestSolution*) <cost(Best of $\mathcal{R}$) and cost(*CurrentSolution*) $>$ cost(*BestSolution*)
  - Perform an intensification phase on *BestSolution*
  - Add *BestSolution* to $\mathcal{R}$ and go to 1

Figure 6: Main Path Relinking Loop

It is clear that the path relinking approach induces a trajectory not easily accomplished by the initial tabu search procedure. This claim is supported by the results reported in the next section, which also tend to show that improved solutions are found by using the path relinking procedure we propose.

4 Experimental results

The objectives of the experiments is to calibrate and analyze the proposed path relinking procedure.

Other than identifying appropriate values for a few key parameters (Section 4.1), the calibration phase aims explore the impact on the solution quality of each combination of strategies to build the reference set (Section 3.1) and select initial and guiding solutions (Section 3.2). We present these results in Section 4.2, while the influence of randomness is examined in Section 4.3. Experimental analyses as well as comparisons with the cycle-based tabu search and branch-and-bound are provided in Section 4.4.

The computer code is written in C++. The exact evaluation of the capacitated multi-commodity network flow problems is done using the linear programming solver of CPLEX 6.5 (1999). All tests were conducted on one 400MHz processor of a 64-processors Sun Enterprise 10000 with 64 Gigabyte of RAM, operating under Solaris 2.7. Computing times are reported in CPU seconds.

4.1 Parameter settings

Two parameters have to be calibrated: the cardinality of the reference set $|\mathcal{R}|$ and the criteria to launch a path relinking phase. Three values, 6, 8, and 15, were tested for $|\mathcal{R}|$. The path relinking phase is started will after a number of iterations of the tabu search procedure without improvement of the best value of the objective function. We retained 10, 20, and 50 iterations for this parameter, *PRstart*. Thus, in total, 9 set of parameters were be tested.

Strategies *S3* and *C1*, for building the reference set and selecting initial and guiding solutions, respectively, were used for these tests. Experiments were conducted on 10 problems covering networks from 100 to 700 design arcs and from 10 to 400 commodities. For more details on these problems, refer to Ghamlouche, Crainic, and Gendreau (2001).

For each parameter set, we run the path relinking algorithm 3 times on each problem. The average gap for each problem was calculated as the average of the gaps between the path relinking solutions and the best known solution for the same problem (that of the branch-and-bound procedure of CPLEX 6.5, when available, or that obtained by Ghamlouche, Crainic, and Gendreau 2001, otherwise). The average gap assigned to each parameter set is the

average of the average gaps of all problems. Moreover, we assign to each problem a score from 9 to 1 to each parameter set. This score depends on the value of the average gap and is determined as follows: For each problem, a score of 9 is assigned to the parameter set giving the lowest average gap over 3 runs, a score of 8 is assigned to the parameter set giving the second lowest average gap, and so on. The total score assigned to a parameter set is the sum of scores assigned to this set for each problem. Average gaps and scores for each parameter set are summarized in Table 1.

$\|\mathcal{R}\|$, $PRstart$	Av. Gap	score
6, 10	2.07%	60
8, 10	2.90%	39
15, 10	3.30%	28
6, 20	1.42%	75
8, 20	2.04%	53
15, 20	2.62%	42
6, 50	1.91%	55
8, 50	1.95%	64
15, 50	2.20%	49

Table 1: Parameter Set Performances

The results displayed in Table 1 seem to indicate that increasing the dimension of the reference set is not necessarily beneficial. In fact, the results point to the parameter set $|\mathcal{R}| = 6$ and $PRstart = 20$ as the most effective: it offers the lowest average gap and the highest score. This setting was used in all the experiments reported in the rest of this section.

4.2 Path Relinking Strategies

We tested all combination of strategies in order to identify the best way to build the reference set (strategy S_i) and select initial and guiding solutions (strategy C_j). The best (S_i, C_j) combination was then used for the extensive experimental analysis of the path relinking procedure.

The same 10 problems used to calibrate the parameter settings were also used here. Moreover, each run is repeated 3 times. Thus, since there are 36 possible combinations of strategies, a total of 1080 run were performed. All strategies have been run for the same number of iterations. Therefore, the performance of each combination of strategies is measured as the average improvement in solution quality, compared to the corresponding result of the cycle-based tabu search.

The comparative performance of all combination of strategies is presented in Table 2. The first conclusion is that path relinking offers a consistenlty better performance in terms of solution quality than the local search heuristic. All combinations of strategies, except one,

	C1	*C2*	*C3*	*C4*	*C(5)*	*C(6)*
S1	-0.51%	-0.91%	-0.84%	-0.47%	-0.55%	-0.42%
S2	-0.23%	-0.18%	-0.73%	-0.39%	-0.50%	-0.19%
S3	-0.54%	-0.51%	-0.93%	-0.69%	-0.99%	-0.80%
S4	-0.39%	-0.80%	-0.75%	-0.39%	-0.59%	-0.29%
S5	-0.36%	-0.69%	-0.61%	-0.41%	-0.45%	-0.43%
S6	-0.46%	-0.46%	-0.01%	-0.47%	0.08%	-0.16%

Table 2: Average Improvement Over Cycle-based Tabu Search

improve the results of cycle-based tabu search.

Table 2 identifies strategies *S3* and *C5* as offering the best results. This set was therefore retained for our experimental analyses.

The combination (*S3*, *C5*) offers a combination of very good solutions obtained over time - local optima better than the current best in $\mathcal{R}$ - and a possibly long exploration that proceeds towards the best solution in the reference set while atarting from the solution most different from it. It is noteworthy that the second best couple of strategies, (*S3*, *C3*) is of a very similar structure. The success of these combinations confirms the importance of, on the one hand, the presence of elite solutions in the reference set and, on the other hand, of relinking strategies that diversify the solutions considered. This emphasizes the diversification effect of path relinking and supports some of the conclusions of Laguna and Armentano (2001).

It is interesting to examine the case of strategy *S1*. One would expect this strategy, which selects overall best solutions, to perform very well. This is not generally the case, however. One explanation may actually come from their very nature of overall best solutions generated by a local search procedure combined to the limited number of solutions included in $\mathcal{R}$. These solutions may share "too" many characteristics and not offer sufficient room for diversification. It is indeed noteworthy that the best performance of strategy *S1* is obtained jointly with strategy *C2* that builds a path from the second best to the best solution in $\mathcal{R}$. This couple, the third best overall, thus implements an enhanced intensification phase around the best solutions found by the local search.

The previous remarks underscore the capability of path relinking to implement both intensification and diversification phases. They also emphasize the need for further studies regarding the behaviour of strategies to to build the reference set and to select the solutions to be linked. These studies are linked to those regarding scatter search (Glover 1994, Glover and

Laguna 1997, Glover, Laguna, and Marti 2000) and other population-based meta-heuristics and are clearly beyond the scope of this paper.

4.3 Influence of Sampling

We desire to investigate the impact of this element of randomness on the robustness of the cycle-based tabu search and path relinking procedures.

In general, one uses sampling in heuristic algorithms to save computation time. The corresponding price is that solutions produced in different runs are generally different. Relatively large differences usually signal a significant impact of the random factors and a somewhat less robust method. The solution of choice then is to perform multiple runs, to select the best solution. Of course, the corresponding running time of this strategy is the total computing time of all runs, which may rapidly become very onerous. On the other hand, when differences are small enough, the first solution that is produced may be considered as representative and "the" best solution. The algorithm may then be considered robust and not sensitive to sampling.

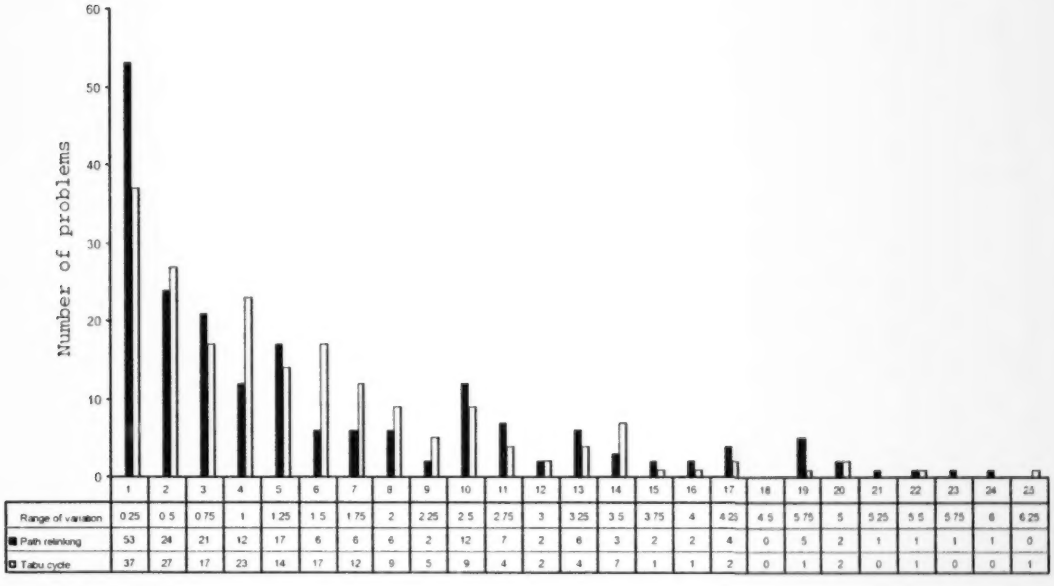
We run both the path relinking and the cycle-based tabu search meta-heuristics 3 times on each of the 196 problems used for experimentation. For each problem, the relative gap between solutions produced over the 3 runs is computed as the percentage gap between the worst and the best solution relatively to the worst solution.

Figure 7 displays the relative gap distributions for the two procedures. Relative gaps are grouped within intervals of 0.25 points of percentage. In each interval, the black and white columns are associated to the path relinking and tabu search, respectively, and represent the number of problems with results in the specific range. Two conclusions emerge from the figures displayed in Figure 7. First, the relative gaps for both meta-heuristics are tight and the methods appear robust. In fact 74% of the problems solved with both meta-heuristics present a relative gap less than or equal to 2%. Second, path relinking is even less influenced by sampling and more robust. This follows from the fact that sampling is replaced during the path relinking phase by a deterministic way to select the candidate links in $\mathcal{C}$.

4.4 Result Analysis

To evaluate the performance of the path relinking algorithm proposed in this paper, we compare its output to the results of the cycle-based tabu search (Ghamlouche, Crainic, and Gendreau 2001) and to the optimal solutions obtained using the branch-and-bound algorithm of CPLEX 6.5 (1999).

The same two data sets of networks used by Ghamlouche, Crainic, and Gendreau (2001)



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
Range of variation	0.25	0.5	0.75	1	1.25	1.5	1.75	2	2.25	2.5	2.75	3	3.25	3.5	3.75	4	4.25	4.5	5.75	5	5.25	5.5	5.75	6	6.25
Path relinking	53	24	21	12	17	6	6	6	2	12	7	2	6	3	2	2	4	0	5	2	1	1	1	1	0
Tabu cycle	37	27	17	23	14	17	12	9	5	9	4	2	4	7	1	1	2	0	1	2	0	1	0	0	1

Figure 7: Influence of sampling

were also used to test the path relinking method. Problems in both sets are general transshipment networks with no parallel arcs. Each commodity corresponds to a single origin-destination pair. On each arc, routing costs are the same for all commodities. Problem instances have been generated to offer for each network size a variety of fixed cost to routing cost ratios and capacity to demand ratios. detailed description of problem instances is given in Crainic, Frangioni, and Gendron (2001; see also Gendron and Crainic 1994, 1996). The problem generators as well as the problem instances can be obtained from the authors.

The first set of problem instances, denoted **C**, contains 43 problems defined by the number of nodes (20, 25, 30, 100), arcs (100, 230, 300, 400, 520, 700), and commodities (10, 30, 40, 100, 200, 400), as well as two letters summarizing the fixed cost and capacity information: a relatively high or low fixed cost relative to the routing cost is signaled by the letter **F** or **V**, respectively, while letters **T** and **L** indicate respectively if the problem is tightly or somewhat loosely capacitated compared to the total demand. Detailed results for this set are presented in this section.

Tables 3 and 4 display results for set **C**. Problems are identified in the first column by the number of nodes, arcs, and commodities, as well as two letters summarizing the fixed cost and capacity information. The OPT column corresponds to the solution of the branch-and-bound algorithm solved using CPLEX 6.5 (1999) on the same computer. A limit of 10 hours was imposed. An **X** indicates that the procedure has failed to produce a feasible solution within this time limit, while a **t** indicates that the procedure stopped due to a time limit condition.

To facilitate comparisons in terms of solution quality and computation time, the cycle-based tabu search and the path relinking algorithm were run 3 times for each problem. Columns TC and AV.TC hold respectively the best and the average solution of the cycle-based tabu search, while columns PR and AV.PR display the best and the average solution found by the path relinking approach, respectively. The figures in parentheses in the first tree columns represent total computation times in CPU seconds. The corresponding figures in columns AV.TC and AV.PR represent the average total computation times in CPU seconds. Column IMPROV displays the percentage of improvement of the average path relinking solution relative to that of the cycle-based tabu search (i.e AV.TC).

Two main observations emerge from the analysis of results presented in Tables 3 and 4. First, the path relinking procedures displays a satisfactory behavior. Comparing columns TC and PR, one observes that path relinking obtains the optimal solutions for 6 problems compared to 2 for TC. Moreover, with very few exceptions, the proposed procedure improves the best solutions found by the cycle-based tabu search. The same behavior is also observed when average solution values are compared: The overall performance is clearly superior for the path relinking approach. The performance of path relinking also appears consistent over the range of problems, including relatively large ones in terms of number of arcs (e.g., 700) and commodities (e.g., 400). When branch-and-bound failed to identify a feasible solution within 10 hours CPU time, the proposed algorithm has identified best-known solutions for these problems with a reasonable computational effort.

PROB	OPT	TC	PR	AV. TC	AV. PR	IMPROV
25,100,10,V,L	14712	14712	14712	14724.67	14712	-0.09%
	(0.36)	(19.08)	(12.97)	(19.38)	(13.01)	
25,100,10,F,L	14941	14941	14941	15216.33	15314	0.64%
	(53.64)	(22.55)	(15.2)	(22.80)	(16.3)	
25,100,10,F,T	49899	50622	49899	50759.00	50192.66	-1.12%
	(40.58)	(31.39)	(22.1)	(32.10)	(24.5)	
25,100,30,V,T	365272	365385	365322	365385	365323.66	-0.02%
	(16.62)	(121.30)	(91.9)	(123.98)	(93.2)	
25,100,30,F,L	37055	38901	37928	39120.27	38043.33	-2.75%
	(1727.96)	(102.21)	(72.57)	(106.33)	(79.5)	
25,100,30,F,T	85530	86732	86215	86944.00	86781.66	-0.19%
	(534.18)	(123.88)	(97.78)	(125.69)	(99.67)	
20,230,40,V,L	423848	425091	424385	425272.67	424620.33	-0.15%
	(10.43)	(217.29)	(129.46)	(222.09)	(132.3)	
20,230,40,V,T	371475	371893	371475	372287	371553.66	-0.20%
	(52.37)	(213.64)	(141.4)	(230.84)	(149.3)	
20,230,40,F,T	643036	643774	644464	645644.67	645070.33	-0.09%
	(671.76)	(242.68)	(146.1)	(249.24)	(146.3)	
20,300,40,V,L	429398	429535	429398	429736.33	429443.66	-0.07%
	(3.76)	(323.78)	(212.73)	(332.67)	(247.32)	
20,300,40,F,L	586077	592043	593304	597613	594944.66	-0.45%
	(145.55)	(340.95)	(223.87)	(347.16)	(240.6)	
20,300,40,V,T	464509	464767	464509	465884.67	464564	-0.28%
	(83.88)	(349.53)	(213.74)	(370.99)	(246.35)	
20,300,40,F,T	604198	609511	604828	610782.0	609259.66	-0.25%
	(59.88)	(352.29)	(137.28)	(364.72)	(138.45)	
20,230,200,V,L	94386	100620	100205	102042.67	101119.33	-0.90%
	(t)	(2477.34)	(2017.35)	(2606.79)	(2034.54)	
20,230,200,F,L	141737.4	149715	148044	150493.33	149536.33	-0.64%
	(t)	(2943.76)	(2493.49)	(3158.22)	(2507.5)	
20,230,200,V,T	97914	103945	104128	105448.00	105730.66	0.27%
	(t)	(2016.35)	(1923.25)	(2282.03)	(1945.67)	
20,230,200,F,T	137271	147385	144857	147815.33	147543	-0.18%
	(t)	(3601.90)	(2926.96)	(3868.50)	(2954.46)	
20,300,200,V,L	74972.4	80266	79128.5	81011.00	79926.5	-1.34%
	(t)	(4247.14)	(3487.2)	(3974.91)	(3560.5)	
20,300,200,F,L	117306	122072	123179	125292.67	125412	0.10%
	(t)	(3267.94)	(3765.34)	(4524.49)	(3912.5)	
20,300,200,V,T	74991	79600	77839.5	80147.33	79253.5	-1.12%
	(t)	(4235.15)	(3691.24)	(4396.22)	(3860.3)	
20,300,200,F,T	108252	116106	111590	116650.67	116251.66	-0.34%
	(t)	(4657.54)	(3546.58)	(4906.50)	(4001.23)	

Table 3: Computational Results, **C** problems

PROB	OPT	TC	PR	AV.TC	AV.PR	IMPROV
100,400,10,V,L	28423	28677	28485	28779	28579.66	-0.69%
	(84.81)	(208.58)	(95.97)	(225.12)	(98.9)	
100,400,10,F,L	24436	24022	24244	24045.33	24316	1.13%
	(t)	(178.52)	(109.66)	(191.07)	(112.45)	
100,400,10,F,T	66364	66802	64527	68215.00	65153	-4.49%
	(t)	(425.68)	(193.4)	(432.01)	(201.34)	
100,400,30,V,T	385544	385508	384879	385510.33	385166	-0.09%
	(t)	(1161.06)	(424.08)	(1169.70)	(450.76)	
100,400,30,F,L	50496	51070	51187	52078.33	51499.33	-1.11%
	(t)	(630.10)	(297.08)	(648.47)	(301.4)	
100,400,30,F,T	141278	145334	141548	146529.33	143677.66	-1.95%
	(t)	(1208.95)	(529.02)	(1235.27)	(579.32)	
30,520,100,V,L	53966	54877	54282	55097.67	54519.33	-1.05%
	(t)	(2416.02)	(1250.59)	(2438.52)	(1351.4)	
30,520,100,F,L	95294	100646	102016	101966.33	103865	1.86%
	(t)	(2344.99)	(1722.43)	(2365.45)	(1843.2)	
30,520,100,V,T	52085	52786	52510	53214.67	52563.33	-1.22%
	(t)	(2601.53)	(1381.43)	(2643.85)	(1423.2)	
30,520,100,F,T	98357	104346	104188	105282.33	104426	-0.81%
	(t)	(2344.76)	(1336.9)	(2418.28)	(1370.5)	
30,700,100,V,L	47603	48553	47924	48914.00	48078.33	-1.71%
	(1736.05)	(3064.96)	(1730.15)	(3097.60)	(1898.98)	
30,700,100,F,L	60525	62611	62349	62953.33	63364.66	0.65%
	(t)	(3083)	(2084.08)	(3153.04)	(2189.6)	
30,700,100,V,T	45944.5	46685	46565	46889.33	46730	-0.34%
	(t)	(3293.08)	(1633.94)	(3388.79)	(1674.09)	
30,700,100,F,T	55709	56534	56246	56895.67	56687.66	-0.37%
	(t)	(3219.31)	(1534.6)	(3316.46)	(1765.3)	
30,520,400,V,L	112997.5	120652	118873	120499.33	119248	-1.04%
	(t)	(36530.8)	(29650.7)	(46875.30)	(31546.23)	
30,520,400,F,L	X	161098	159007	163226.00	162107.66	-0.69%
	(t)	(429296)	(33641.2)	(40922.57)	(35671.23)	
30,520,400,V,T	X	121588	119744	122570.00	119847	-2.22%
	(t)	(28214)	(20461.9)	(33018.83)	(23546.4)	
30,520,400,F,T	X	167939	163352	167301.33	165482.33	-1.09%
	(t)	(40010.9)	(59820.1)	(49476.67)	(60123.3)	
30,700,400,V,L	X	104860	103138	105362.67	103521.33	-1.75%
	(t)	(26332)	(17881.3)	(27681.63)	(19433.4)	
30,700,400,F,L	X	149583	144615	151336.33	145998.33	-3.53%
	(t)	(55030.8)	(57574.8)	(63170.20)	(58761.8)	
30,700,400,V,T	X	101623	99606	101822.33	99837.2	-1.95%
	(t)	(36226.3)	(31405.7)	(42725.67)	(32450.2)	
30,700,400,F,T	X	143324	138777	144634.00	140891.66	-2.59%
	(t)	(51877.9)	(48693.7)	(58196.33)	(51234.5)	

Table 4: Computational Results, **C** problems

The second observation concerns the computing effort. Indeed, despite of the fact that the path relinking procedure is stopped on the same criterion as the cycle-based tabu search, 400 iterations, Tables 3 and 4 indicate that the computational time for the former is often significantly shorter compared to that of the latter. This follows principally from the reduced number of resolutions of the multicommodity capacitated network flow problem and the fact that the dimension of the neighbourhood used during the path relinking phase is smaller.

The interesting performance and behavior of the proposed path relinking procedure is confirmed by the results obtained on the second set of problems instances, denoted **R**. There are 116 problems divided in 18 groups. Each group contains the same number of nodes, arcs, and commodities but with different combined level of fixed cost and capacity ratios. Three levels of fixed cost and capacity ratios are considered: F01 = 0.01, F05 = 0.05, and F10 = 0.10 indicating continuously higher levels of fixed costs compared to routing costs, and C1 = 1, C2 = 2, and C8 = 8 that signal that the total capacity available becomes increasingly tighter relatively to the total demand (the formulas used to compute these ratios are presented in the Appendix). Thus, the easiest problems generally have F01 and C1 ratios, while the most difficult ones have F10 and C8 characteristics. Only aggregated statistics are used in the following to support the discussion. Detailed results are included in the Appendix.

Table 5 displays the distribution of the optimality gaps relative to CPLEX branch-and-bound for both the cycle-based tabu search and the path relinking procedures. Values are calculated for both the best and average solutions for the C and R problems. Column PSet identifies the problems set, the X column indicates the number of problems where branch-and-bound did not find a feasible solution (in 10 hours), the OPT column indicates the number of optimal solutions found by the meta-heuristics, and the IMP column displays the number of problems where the meta-heuristic solution is better than the one found by branch-and-bound after 10 hours of computation. The next six columns correspond to the indicated gap intervals.

	PSet	X	OPT	IMP	(0-2%]	(2-4%]	(4-6%]	(6-8%]	(8-10%]	>10%
TC	C	7	2	2	18	2	4	8	-	-
PR	C	7	6	3	14	4	6	3	-	-
TC	R	-	23	-	48	31	20	14	11	6
PR	R	-	38	-	56	26	14	11	4	4
AV.TC	C	7	-	2	13	8	2	9	2	-
AV.PR	C	7	1	3	18	2	4	7	1	-
AV.TC	R	-	19	-	43	27	25	17	10	12
AV.PR	R	-	20	-	61	29	17	10	8	8

Table 5: Distribution of Relative Gaps

The results displayed in Table 5 show that path relinking obtains a larger number of optimal solutions than the cycle-based tabu search. Moreover, the distribution of the gaps indicates that path relinking solutions are closer to the optimum than those of the cycle-based tabu search, in terms of both best and average solution values On average, for problems

in sets **C** and **R**, path relinking obtains gaps of 2.29% and 3.08%, respectively, of the best solutions found by branch-and-bound. But then, branch-and-bound cannot address all problems. Compared to the cycle-based tabu search, the average improvement obtained by path relinking is 0.80% and 0.83% for problems in sets **C** and **R**, respectively, while the maximum improvement is 4.49% and 4.75%, respectively.

	C1		C2		C8	
	TC	PR	TC	PR	TC	PR
F01	1.33%	0.76%	1.39%	0.78%	1.91%	1.15%
F05	3.32%	2.43%	3 65%	2.64%	4.43%	3.23%
F10	4.15%	3.09%	4.35%	3.04%	5.25%	4.11%
	C1		C2		C8	
	AV.TC	AV.PR	AV.TC	AV.PR	AV.TC	AV.PR
F01	1.69%	0.99%	1.70%	1.00%	2.17%	1.44%
F05	4.20%	3.27%	4.20%	3.49%	5.20%	4.15%
F10	5.00%	4.05%	5.36%	4.14%	6.32%	5.50%

Table 6: Gap Distribution According to Fixed Cost and Capacity Level, **R** Problems

$\|P\|,\|N\|,\|A\|$	TC	PR	$\|P\|,\|N\|,\|A\|$	TC	PR	$\|P\|,\|N\|,\|A\|$	TC	PR
10,10,25	0.00%	0.00%	10,10,50	0.23%	0.08%	10,10,75	0.70%	0.04%
25,10,25	1.41%	0.23%	25,10,50	0.82%	0.36%	25,10,75	1.92%	0.41%
50,10,25	1.48%	0.61%	50,10,50	2.36%	1.14%	50,10,75	2.69%	1.52%
40,20,100	2.50%	1.37%	40,20,200	4.37%	3.59%	40,20,300	3.91%	2.08%
100,20,100	2.98%	2.05%	100,20,200	5.36%	4.93%	100,20,300	6.99%	4.68%
200,20,100	4.80%	4.55%	200,20,200	6.76%	5.41%	200,20,300	7.39%	6.84%
$\|P\|,\|N\|,\|A\|$	AV.TC	AV.PR	$\|P\|,\|N\|,\|A\|$	AV.TC	AV.PR	$\|P\|,\|N\|,\|A\|$	AV.TC	AV.PR
10,10,25	0.05%	0.00%	10,10,50	0.36%	0.12%	10,10,75	0.89%	0.46%
25,10,25	1.70%	0.60%	25,10,50	1.16%	0.55%	25,10,75	2.47%	0.92%
50,10,25	1.92%	0.72%	50,10,50	3.06%	1.93%	50,10,75	3.37%	2.30%
40,20,100	3.34%	1.70%	40,20,200	5.52%	4.84%	40,20,300	4.80%	3.91%
100,20,100	3.52%	2.82%	100,20,200	6.47%	5.90%	100,20,300	7.82%	5.79%
200,20,100	5.29%	4.86%	200,20,200	7.30%	6.42%	200,20,300	9.32%	8.97%

Table 7: Gap Distribution According to Problem Dimensions, **R** Problems

Table 6 displays the optimality gap distribution of the best and average solutions of the cycle-based tabu search and path relinking according to fixed cost and capacity ratios for problems in set **R**. Table 7 displays the same information but according to problem dimensions. The results displayed in these tables underscore the important role of the path relinking phase in the search. In all cases, the path relinking procedure achieves better results than the cycle-based tabu search and reduces significantly the optimality gap. This

behavior is consistent over the entire range of problem dimensions, number of commodities, and variation of fixed cost and capacity. Consequently, we believe that the approach propose performs well and is able to handle variation in problem dimensions and structure.

5 Conclusion

We have presented a path relinking algorithm for the fixed charge, capacitated, multicommodity network design problem. Elite solutions are constructed by performing a simple tabu search algorithm using cycle-based neighbourhood structures. A restricted version of the same cycle-based neighbourhoods is used to build the path linking two elite solutions in the reference set. We have proposed and compared several strategies to build the reference set and for to select initial and guiding solutions. The performance of the algorithm was evaluated using 196 problem instances of several types and sizes.

The results are very satisfactory. The basic idea of combining the cycle-based tabu search and path relinking is a successful one. In most cases, the path relinking achieves better results. Moreover, the algorithm appears robust in terms of solution quality and computational effort. At our knowledge, in fact, the proposed path relinking meta-heuristic offers the best current performance among approximate solution methods for the fixed charge, capacitated, multicommodity network design problem.

The experimental results have also pointed out a number of interesting research perspectives. One such avenue is concerned with the criteria used to select and combine solutions in population-based meta-heuristics, such as path relinking, scatter search, and more traditional genetic algorithms. This issue is especially important when solutions display a mixed integer nature with complex network structures and sub-problems, as is the case for the problems we address. A second research direction concerns the mechanisms to learn (memories) and guide the various phases of the search - local search, diversification, intensification, and so on -, the relations to the already-mentioned population-based methods, and the combinations of these various elements into a comprehensive meta-heuristic for the fixed charge, capacitated, multicommodity network design problem. We intend to report development on both subjects in the relatively close future.

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Appendix

Problems **R** were generated to facilitate the study of the impact of particular problem characteristics on algorithmic performance. The number of nodes, design arcs, and commodities are systematically varied, one at a time, and for each of these networks, nine problem instances were created by combining three levels of fixed cost ratios and three levels of capacity ratios.

The *fixed cost ratio* is computed as $|\mathcal{P}| \sum_{(ij)\in\mathcal{A}} f_{ij} / \sum_{p\in\mathcal{P}} w^p \sum_{(ij)\in\mathcal{A}} c_{ij}^p$, and the three values considered are F01 = 0.01, F05 = 0.05, and F10 = 0.10 corresponding to continuously higher levels of fixed costs compared to the routing costs. The *capacity ratio* is is computed as $|\mathcal{A}| \sum_{p\in\mathcal{P}} w^p / \sum_{(ij)\in\mathcal{A}} u_{ij}$, and the values considered are C1 = 1, C2 = 2, and C8 = 8, indicating that the total capacity available becomes increasingly tighter relatively to the total demand. Thus, the easiest problems generally have F01 and C1 ratios, while the most difficult ones have F10 and C8 characteristics.

Results for each problem instance of set **R** are given in Tables 8 to 25. The following information is displayed:

- OPT: The optimal solution obtained by using the branch-and-bound algorithm of CPLEX version 6.5. A (t) indicates that the procedure stopped due to a time limit condition. A limit of 10 hours CPU time was imposed.
- TC and PR: The best solution obtained respectively by the cycle-based tabu search and path relinking over 3 runs.
- AV.TC and AV.PR: The average solution obtained respectively by the cycle-based tabu search and path relinking over 3 runs.
- IMPROV: The average improvement in solution quality between the average solution of path relinking and that of the cycle-based tabu search.

All tests were conducted on one 400MHz processor of a 64-processors Sun Enterprise 10000 with 64 Gigabyte of RAM, operating under Solaris 2.7.

The figures in parentheses in the first 3 columns represent total computation times in CPU seconds, while the corresponding figures in columns AV.TC and AV.PR represent average total computation times.

Table 8: Computational Results R01: 10,25,10

PROB	OPT	TC	PR	AV.TC	AV.PR	IMPROV
R01,F01,C1	74079	74079	74079	74079	74079	0%
	(0.04)	(6.53)	(6.66)	(6.66)	(6.43)	
R01,F05,C1	92403	92403	92403	92403	92403	0%
	(0.04)	(6.13)	(4.95)	(6.37)	(6.13)	
R01,F10,C1	115304	115304	115304	115304	115304	0%
	(0.09)	(6.28)	(4.76)	(6.40)	(5.32)	
R01,F01,C2	84908	84908	84908	85066.67	84908	-0.19%
	(0.46)	(7.62)	(6.39)	(8.70)	(6.34)	
R01,F05,C2	113036	113036	113036	113036	113036	0%
	(1.35)	(9.84)	(7.42)	(9.36)	(7.45)	
R01,F10,C2	147599	147599	147599	147796.67	147599	-0.13%
	(2.36)	(8.62)	(7.74)	(9.16)	(7.84)	

Table 9: Computational Results R02: 10,25,25

PROB	OPT	TC	PR	AV.TC	AV.PR	IMPROV
R02,F01,C1	232239	232239	232239	232239	232239	0%
	(0.31)	(16.21)	(13.03)	(17.48)	(14.76)	
R02,F05,C1	322453	328005	326597	330429.00	327066.33	-1.02%
	(1.12)	(24.67)	(20.09)	(24.89)	(23.56)	
R02,F10,C1	419503	439687	419503	440508.00	426999.66	-3.07%
	(1.37)	(25.31)	(20.69)	(25.51)	(26.43)	
R02,F01,C2	316437	316437	316437	316437.00	316437	0%
	(0.23)	(25.55)	(21.92)	(25.86)	(22.12)	
R02,F05,C2	431250	432234	431734	433979.00	432470	-0.35%
	(0.53)	(29.45)	(23.28)	(29.85)	(26.34)	
R02,F10,C2	559578	569032	559578	571400.67	560082.33	-1.98%
	(0.55)	(27.32)	(23.56)	(29.06)	(24.56)	

Table 10: Computational Results R03:10,25,50

PROB	OPT	TC	PR	AV.TC	AV.PR	IMPROV
R03,F01,C1	484830	484830	484830	484830	485678.33	0.17%
	(0.67)	(52.46)	(58.16)	(53.26)	(54.23)	
R03,F05,C1	703362	716524	704893	718578.33	706213	-1.72%
	(2.01)	(60.65)	(58.17)	(66.84)	(60.23)	
R03,F10,C1	944990	985909	962408	992413.67	964399	-2.82%
	(2.34)	(75.81)	(58.25)	(76.22)	(60.12)	
R03,F01,C2	704247	705207	705207	705207.00	705207	0%
	(0.47)	(89.35)	(46.86)	(89.88)	(50.23)	
R03,F05,C2	932897	942929	939055	943995.00	939372.66	-0.49%
	(1.26)	(89.41)	(67.14)	(90.34)	(70.23)	
R03,F10,C2	1188638	1205740	1197950	1224473.33	1198550	-2.12%
	(0.98)	(90.14)	(60.16)	(91.32)	(64.12)	

Table 11: Computational Results R04:10,50,10

PROB	OPT	TC	PR	AV.TC	AV.PR	IMPROV
R04,F01,C1	31730	31730	31730	31730	31730	0%
	(0.02)	(9.51)	(6.84)	(9.59)	(7.4)	
R04,F05,C1	48920	48920	48920	48920	48920	0%
	(0.06)	(8.85)	(6.28)	(9.52)	(7.34)	
R04,F10,C1	63767	63767	63767	63767	63767	0%
	(0.06)	(9.75)	(7.36)	(10.15)	(8.43)	
R04,F01,C2	33740	33740	33740	33740	33740	0%
	(0.57)	(9.85)	(7.4)	(10.19)	(876)	
R04,F05,C2	53790	53790	53790	53790	53790	0%
	(2.62)	(10.39)	(7.62)	(10.65)	(7.45)	
R04,F10,C2	74030	74030	74030	74030	74030	0%
	(1.72)	(10.37)	(8.18)	(10.76)	(8.46)	
R04,F01,C8	68291.7	68517	68310	68551.67	68456	-0.14%
	(5.14)	(13.27)	(11.45)	(14.53)	(12.53)	
R04,F05,C8	113004	113226	113226	113744	113226	-0.46%
	(9.41)	(15.79)	(11.69)	(15.99)	(12.56)	
R04,F10,C8	163208	165657	164091	166764.67	164317	-1.47%
	(7.61)	(15.95)	(11.54)	(16.27)	(13.34)	

Table 12: Computational Results R05:10,50,25

PROB	OPT	TC	PR	AV.TC	AV.PR	IMPROV
R05,F01,C1	123003	123109	123003	123465.67	123199.66	-0.22%
	(0.38)	(23.48)	(17.94)	(23.61)	(20.21)	
R05,F05,C1	170060	172517	170467	173327.67	171573	-1.01%
	(1.51)	(24.24)	(18.4)	(24.44)	(18.45)	
R05,F10,C1	221486	225950	221486	226440.00	221967.33	-1.98%
	(8.44)	(25.57)	(19.78)	(25.70)	(20.4)	
R05,F01,C2	131608	131888	131608	132153.33	131662.66	-0.37%
	(1.58)	(26.55)	(20.38)	(26.89)	(20.67)	
R05,F05,C2	204157	206497	205764	207021.33	205764	-0.61%
	(15.60)	(28.49)	(21.53)	(28.90)	(23.43)	
R05,F10,C2	286524	290574	290574	293937.67	292887.66	-0.36%
	(65.12)	(28.57)	(22.9)	(30.12)	(22.8)	
R05,F01,C8	278372	278372	278372	278372	278372	0%
	(1.45)	(43.70)	(47.25)	(52.08)	(49.32)	
R05,F05,C8	445810	448542	449133	449319.00	446986.33	-0.52%
	(1.99)	(50.45)	(45.83)	(53.83)	(47.9)	
R05,F10,C8	625879	628628	626291	630216.00	628123.66	-0.33%
	(2.20)	(42.50)	(39.65)	(46.58)	(40.45)	

Table 13: Computational Results R06:10,50,50

PROB	OPT	TC	PR	AV.TC	AV.PR	IMPROV
R06,F01,C1	245936	247829	246365	248527.67	247028.66	-0.60%
	(3.97)	(58.64)	(48.84)	(59.55)	(49.87)	
R06,F05,C1	401685	410457	405672	414565.67	407483.66	-1.71%
	(51.12)	(71.69)	(57.08)	(72.96)	(60.12)	
R06,F10,C1	559477	587827	572112	592116.33	576620.33	-2.62%
	(205.18)	(69.56)	(61.74)	(73.04)	(59.24)	
R06,F01,C2	286682	288200	287510	288861.67	287551	-0.45%
	(25.18)	(74.92)	(62.51)	(79.97)	(62.4)	
R06,F05,C2	498266	517614	501560	519212.67	512575.33	-1.28%
	(463.4)	(94.42)	(71.95)	(97.03)	(73.67)	
R06,F10,C2	734414	775792	753337	783014.67	763504	-2.49%
	(1855.94)	(99.10)	(74.51)	(100.03)	(71.08)	
R06,F01,C8	682921	683481	683614	683525.33	683692.66	0.02%
	(1.47)	(172.53	(125.96)	(177.05)	(126.87)	
R06,F05,C8	1030479	1042510	1038600	1044586.67	1041900	-0.26%
	(1.45)	(176.18)	(130.92)	(190.94)	(133.98)	
R06,F10,C8	423316	431329	433690	442105.33	440456.33	-0.37%
	(46.56)	(54.54)	(45.34)	(56.35)	(45.45)	

Table 14: Computational Results R07:10,75,10

PROB	OPT	TC	PR	AV.TC	AV.PR	IMPROV
R07,F01,C1	32807	32807	32807	32807	32807	0%
	(0.08)	(15.65)	(14.00)	(14.99)	(13.6)	
R07,F05,C1	47252	47252	47252	47252	47252	0%
	(0.13)	(12.37)	(8.8)	(12.61)	(9.21)	
R07,F10,C1	62962	62962.0	62962	62962	62987.66	0.04%
	(0.62)	(15.17)	(9.62)	(15.42)	(10.34)	
R07,F01,C2	37432	37432.0	37432	37432	37432	0%
	(0.66)	(13.76)	(10.06)	(13.89)	(10.45)	
R07,F05,C2	56475	56841.0	56475	56841	56552.33	-0.51%
	(4.39)	(15.05)	(10.74)	(15.18)	(11.43)	
R07,F10,C2	77249	77863.0	77249	78842.67	78489.66	-0.45%
	(19.01)	(14.20)	(10.61)	(15.32)	(10.45)	
R07,F01,C8	59947	59947.0	59947	59947	60061.66	0.19%
	(11.05)	(17.99)	(15.03)	(19.62)	(18.62)	
R07,F05,C8	99194	102203.	99502	102747.33	99757.33	-2.91%
	(21.75)	(20.63)	(14.52)	(21.26)	(14.23)	
R07,F10,C8	141692	144329.	141788	144079	143898.33	-0.13%
	(54.39)	(19.16)	(14.84)	(20.62)	(15.23)	

Table 15: Computational Results R08: 10,75,25

PROB	OPT	TC	PR	AV.TC	AV.PR	IMPROV
R08,F01,C1	102531	102556	102531	102880.00	102547.66	-0.32%
	(1.33)	(31.60)	(26.15)	(32.02)	(25.14)	
R08,F05,C1	143894	145434	143894	145818.67	143894	-1.32%
	(0.54)	(31.53)	(24.62)	(31.93)	(24.45)	
R08,F10,C1	182793	182793	182793	182969.00	184068.33	0.60%
	(1.35)	(31.83)	(26.29)	(31.91)	(25.34)	
R08,F01,C2	109325	110193	109325	110308.33	109358.33	-0.86%
	(5.45)	(34.28)	(25.45)	(34.64)	(26.45)	
R08,F05,C2	157047	160848	157901	161447.67	158039	-2.11%
	(34.23)	(35.76)	(27.7)	(36.36)	(28.8)	
R08,F10,C2	207540	214793	208325	214993.00	209766	-2.43%
	(34.23)	(34.52)	(27.95)	(35.79)	(27.7)	
R08,F01,C8	154160	156738	154327	157065.33	155292.33	-1.13%
	(75.38)	(54.46)	(38.12)	(55.01)	(39.13)	
R08,F05,C8	274866.5	282525	278443	287224.33	280858.33	-2.22%
	(404.49)	(53.88)	(42.52)	(55.15)	(41.15)	
R08,F10,C8	415793	436669	421292	444136.00	427881	-3.66%
	(776.56)	(54.40)	(41.33)	(56.67)	(43.8)	

Table 16: Computational Results R09: 10,75,50

PROB	OPT	TC	PR	AV.TC	AV.PR	IMPROV
R09,F01,C1	171512	171902	171572	172382.67	171655.33	-0.42%
	(2.33)	(71.94)	(55.89)	(74.39)	(56.45)	
R09,F05,C1	296712	305497	302448	308971.67	303722	-1.70%
	(32.65)	(66.57)	(66.94)	(76.45)	(66.56)	
R09,F10,C1	424266	435762	431584	440892.33	436532.66	-0.99%
	(218.77)	(73.17)	(64.81)	(80.40)	(65.56)	
R09,F01,C2	192736	193734	192736	194155.67	193203.66	-0.49%
	(10.2)	(80.08)	(67.11)	(83.27)	(69.13)	
R09,F05,C2	357318	368592	366418	373219.67	371407.66	-0.49%
	(143.32)	(90.21)	(76.96)	(99.48)	(79.45)	
R09,F10,C2	522187	555768	533927	560263.67	543955	-2.91%
	(1227.52)	(86.08)	(80.26)	(93.20)	(83.98)	
R09,F01,C8	345057	348732	346773	349684.33	348735.33	-0.27%
	(27.83)	(151.35)	(124.73)	(153.15)	(125.87)	
R09,F05,C8	646579	668296	657541	669379.00	665286.33	-0.61%
	(89.5)	(156.57)	(117.78)	(161.14)	(127.54)	
R09,F10,C8	951136	987258.	979672	992832.00	980308.66	-1.26%
	(310.59)	(155.66)	(123.66)	(159.08)	(133.98)	

Table 17: Computational Results R10: 20,100,40

PROB	OPT	TC	PR	AV.TC	AV.PR	IMPROV
R10,F01,C1	200087	200484	200147	200542.33	200340.33	-0.10%
	(10.72)	(104.95)	(75.22)	(106.74)	(76.34)	
R10,F05,C1	346813.5	350407	347459	352366.00	347835.66	-1.29%
	(788.2)	(110.20)	(85.65)	(116.93)	(88.56)	
R10,F10,C1	488015	502724	502724	505694.00	504601	-0.22%
	(563.29)	(127.87)	(84.8)	(128.99)	(84.56)	
R10,F01,C2	229196	231302	229196	232220.67	229363	-1.23%
	(184.31)	(128.79)	(90.6)	(134.77)	(97.56)	
R10,F05,C2	411664	430254	425230	433229.00	426363.66	-1.58%
	(5123.25)	(149.47)	(102.63)	(149.82)	(110.34)	
R10,F10,C2	609104	646107	628353	664652.33	633081.33	-4.75%
	(27260.53)	(158.59)	(109.82)	(159.23)	(109.54)	
R10,F01,C8	486895	490034	487235	491176.67	487923.33	-0.66%
	(391.33)	(390.26)	(330.31)	(398.56)	(343.89)	
R10,F05,C8	951056	977346	963914	980466.67	967753.33	-1.30%
	(555.52)	(286.08)	(274.29)	(315.74)	(284.76)	
R10,F10,C8	1421740	1469020	1439320	1491893.33	1448780	-2.89%
	(608.38)	(228.92)	(248.12)	(243.37)	(245.78)	

Table 18: Computational Results R11: 20,100,100

PROB	OPT	TC	PR	AV.TC	AV.PR	IMPROV
R11,F01,C1	714431	726585	720236	730686.33	724441	-0.85%
	(392.46)	(441.29)	(355.76)	(447.77)	(358.43)	
R11,F05,C1	1265985	1323620	1296050	1327736.67	1304816.66	-1.73%
	(t)	(489.85)	(510.67)	(601.70)	(528.12)	
R11,F10,C1	1846295	1933560	1940180	1969320	1951226.66	-0.92%
	(t)	(706.75)	(576.94)	(723.43)	(587.76)	
R11,F01,C2	870451	879445	875908	883224.67	877552	-0.64%
	(2960.13)	(819.49)	(550.5[illegible]	(830.33)	(555.43)	
R11,F05,C2	1623640	1715040	1674[illegible]40	1721046.67	1687013.33	-1.98%
	(21739.5)	(1137.94)	(889.13)	(1153.43)	(890.33)	
R11,F10,C2	2414060	2593590	2546730	2600196.67	2606896.66	0.26%
	(t)	(1274.95)	(990.76)	(1332.79)	(1010.45)	
R11,F01,C8	2294912	2294912	2294912	2295296.67	2294912	-0.02%
	(31.51)	(880.22)	(1181.57)	(1088.83)	(1089.56)	
R11,F05,C8	3507100	3540730	3529530	3550923.33	3539613.33	-0.32%
	(79.79)	(1174.59)	(1079.67)	(1235.99)	(1102.99)	
R11,F10,C8	4579353	4616190	4592820	4645180	4653570	0.18%
	(48.51)	(827.84)	(777.86)	(849.09)	(812.45)	

Table 19: Computational Results R12: 20,100,200

PROB	OPT	TC	PR	AV.TC	AV.PR	IMPROV
R12,F01,C1	1639443	1700170	1668370	1706230	1677986.66	-1.66%
	(4436.74)	(2199.46)	(2434.57)	(2720.30)	(2453.3)	
R12,F05,C1	3403074.2	3741660	3717800	3767176.67	3728346.66	-1.03%
	(t)	(3188.77)	(2975.35)	(3477.48)	(3177.3)	
R12,F10,C1	5276171	6055670	6037960	6100926.67	6073666.66	-0.45%
	(t)	(4096.82)	(3245.49)	(4331.77)	(3476.4)	
R12,F01,C2	2303557	2331290	2326930	2332063.33	2325936.66	-0.26%
	(1739.79)	(6504.21)	(5539.91)	(7467.51)	(6432.43)	
R12,F05,C2	4669799	4942920	4954250	4982423.33	4978036.66	-0.09%
	(3145.59)	(7364.80)	(5709.11)	(7574.81)	(5634.98)	
R12,F10,C2	7100019	7562660	7638050	7653836.67	7660990	0.09%
	(1683.84)	(6060.43)	(4735.91)	(7704.07)	(5432.98)	
R12,F01,C8	7635270	7638050	7638050	7638050	7638050	0.00%
	(78.13)	(5336.99)	(4735.91)	(5787.92)	(4654.33)	
R12,F05,C8	10067742	10106800	10085000	10116466.67	10104800	-0.12%
	(61.55)	(3176.64)	(3315.88)	(3347.35)	(3412.5)	
R12,F10,C8	11967768	12064300	12042800	12083133.33	12063866.67	-0.16%
	(50.29)	(2459.84)	(2276.53)	(2503.33)	(2341.4)	

Table 20: Computational Results R13: 20,200,40

PROB	OPT	TC	PR	AV.TC	AV.PR	IMPROV
R13,F01,C1	142947	143778	142947	144437.33	143139.66	-0.90%
	(6.41)	(201.55)	(130.35)	(202.37)	(138.4)	
R13,F05,C1	263800	272019	273853	275961.33	273912	-0.74%
	(547.98)	(203.12)	(141.32)	(205.89)	(145.32)	
R13,F10,C1	365836	375628	378922	379687.67	387201.33	1.98%
	(116.62)	(214.58)	(145.79)	(221.23)	(147.34)	
R13,F01,C2	150977	151875	150977	153080	152617.66	-0.30%
	(90.14)	(208.55)	(143.09)	(209.60)	(141.23)	
R13,F05,C2	282682	290150	289706	293241.67	294079.33	0.29%
	(3551.86)	(225.07)	(142.83)	(231.63)	(144.76)	
R13,F10,C2	406790	430246	435724	435988.33	439691.33	0.85%
	(29695.59)	(246.71)	(159.9)	(252.66)	(164.34)	
R13,F01,C8	208088	213650	210898	214628.67	212134	-1.16%
	(t)	(319.67)	(183.14)	(326.24)	(197.23)	
R13,F05,C8	446997	485018	469035	491599.67	481539.66	-2.05%
	(t)	(379.88)	(182.03)	(383.64)	(189.45)	
R13,F10,C8	698280	787477	761505	801558.67	773893	-3.45%
	(t)	(391.99)	(232.08)	(403.09)	(245.76)	

Table 21: Computational Results R14: 20,200,100

PROB	OPT	TC	PR	AV.TC	AV.PR	IMPROV
R14,F01,C1	403414	416932	410430	418294.67	411263.66	-1.68%
	(495.72)	(561.79)	(413.86)	(567.40)	(412.56)	
R14,F05,C1	750091	775023	778492	792663.67	787719.66	-0.62%
	(t)	(598.24)	(503.86)	(644.45)	(576.32)	
R14,F10,C1	1078595	1133420	1127980	1160773.33	1133240	-2.37%
	(t)	(723.41)	(555.53)	(737.84)	(604.12)	
R14,F01,C2	437607	448725	444170	451845.33	445088.33	-1.50%
	(1327.37)	(633.53)	(463.33)	(637.85)	(475.23)	
R14,F05,C2	852330	890914	883241	897678.00	894028	-0.41%
	(t)	(731.71)	(580.59)	(777.93)	(590.43)	
R14,F10,C2	1230749.7	1309970	1335170	1332676.67	1340806.66	0.61%
	(t)	(818.25)	(608.59)	(833.51)	(673.12)	
R14,F01,C8	670064	698381	689762	701782.33	692596.33	-1.31%
	(t)	(1794.67)	(1294.26)	(2000.69)	(1576.34)	
R14,F05,C8	1643376	1782720	1740600	1791180.00	1771183.33	-1.12%
	(t)	(2516.48)	(1810.25)	(2717.66)	(1965.45)	
R14,F10,C8	2630332	2901140	2939990	2911496.67	3008416.66	3.33%
	(t)	(3223.01	(2555.49)	(3335.62)	(2875.45)	

Table 22: Computational Results R15: 20,200,200

PROB	OPT	TC	PR	AV.TC	AV.PR	IMPROV
R15,F01,C1	1000787	1044180	1027930	1053690	1034716.66	-1.80%
	(5384.72)	(1698.44)	(1361.73)	(1808.50)	(1587.43)	
R15,F05,C1	1979413	2164870	2135640	2183736.67	2183433.33	-0.01%
	(t)	(2323.56)	(1871.08)	(2475.63)	(1896.43)	
R15,F10,C1	2949264	3156520	3131640	3178423.33	3190583.33	0.38%
	(t)	(2546.50)	(2106.42)	(2572.87)	(2474.12)	
R15,F01,C2	1148604	1211560	1184780	1217346.67	1192780	-2.02%
	(t)	(2593.89)	(2049.64)	(2904.63)	(2341.23)	
R15,F05,C2	2488753	2736570	2732310	2747943.33	2771770	0.87%
	(t)	(4999.33)	(5114.44)	(5778.15)	(5045.12)	
R15,F10,C2	3972667	4393650	4264090	4395146.67	4304980	-2.05%
	(t)	(6171.74)	(4594.46)	(6846.45)	(4653.12)	
R15,F01,C8	2301326.6	2355980	2350730	2360816.67	2351980	-0.37%
	(t)	(18162.70)	(14515.9)	(18763.03)	(14875.9)	
R15,F05,C8	5573412.8	5941080	5830430	5944116.67	5858273.33	-1.44%
	(16107.92)	(20972.30)	(23169.4)	(24038.43)	(22453.34)	
R15,F10,C8	8696932	9140060	9123020	9220093.33	9131563.33	-0.96%
	(18930.96)	(14527.20)	(19548.3)	(15536.40)	(18765.76)	

Table 23: Computational Results R16: 20,300,40

PROB	OPT	TC	PR	AV.TC	AV.PR	IMPROV
R16,F01,C1	136161	136851	136161	137553.00	136763.66	-0.57%
	(1.05)	(322.20)	(194.43)	(327.58)	(190.34)	
R16,F05,C1	239500	245003	244070	248182.00	248259.33	0.03%
	(1641.30)	(309.91)	(200.35)	(311.99)	(234.56)	
R16,F10,C1	325671	346724	330138	348585.67	342357.33	-1.79%
	(4345.92)	(306.85)	(212.21)	(312.22)	(240.34)	
R16,F01,C2	138532	140433	139131	141125.33	139996.66	-0.80%
	(27.61)	(327.39)	(184.9)	(328.64)	(203.54)	
R16,F05,C2	241801	250332	248717	253838.67	254805.66	0.38%
	(851.99)	(325.14)	(204.65)	(329.91)	(198.45)	
R16,F10,C2	337762	352450	343165	357835.33	357759	-0.02%
	(8173.47)	(305.98)	(206.89)	(315.81)	(250.45)	
R16,F01,C8	169502	173164	171651	174503.67	172527	-1.13%
	(t)	(382.96)	(209.24)	(387.08)	(278.45)	
R16,F05,C8	352976	374372	371129	376533.67	372825	-0.98%
	(t)	(429.56)	(244.07)	(434.95)	(290.45)	
R16,F10,C8	541626	587242	564151	590970.00	575334	-2.65%
	(t)	(450.30)	(229.92)	(456.95)	(249.45)	

Table 24: Computational Results R17: 20.300,100

PROB	OPT	TC	PR	AV.TC	AV.PR	IMPROV
R17,F01,C1	354138	367923	364325	370314.67	365378.66	-1.33%
	(332.10)	(790.06)	(536.69)	(810.44)	(654.34)	
R17,F05,C1	651025	693482	669935	696950.33	687806.33	-1.31%
	(t)	(822.09)	(710.92)	(866.93)	(806.45)	
R17,F10,C1	917956	990967	973486	996346.00	980134.66	-1.63%
	(t)	(871.10)	(845.3)	(948.55)	(765.52)	
R17,F01,C2	370590	382868	378905	385026.33	380997	-1.05%
	(2258.82)	(866.99)	(615.05)	(899.28)	(654.9)	
R17,F05,C2	708698	761476	743173	768713.67	748048.66	-2.69%
	(t)	(988.79)	(713.2)	(1026.86)	(750.54)	
R17,F10,C2	1029242	1104380	1084280	1116650.00	1098420	-1.63%
	(t)	(1092.96)	(832.08)	(1134.30)	(965.87)	
R17,F01,C8	505310	530313	521468	532371.00	523786	-1.61%
	(t)	(1622.41)	(1026.25)	(1691.09)	(1087.65)	
R17,F05,C8	1112076	1249440	1191580	1265813.33	1209456.6	-4.45%
	(t)	(2313.01)	(1486.98)	(2774.21)	(1643.98)	
R17,F10,C8	1847334	2017720	1985220	2035663.33	2014533.3	-1.04%
	(t)	(3048.11)	(2300.72)	(3165.96)	(2654.43)	

Table 25: Computational Results R18: 20,300,200

PROB	OPT	TC	PR	AV.TC	AV.PR	IMPROV
R18,F01,C1	828559	866838	858043	877666.67	861656.66	-1.82%
	(t)	(2167.16)	(1926.27)	(2235.21)	(2076.54)	
R18,F05,C1	1542197	1671810	1649730	1726433.33	1712110	-0.83%
	(t)	(2609.61)	(2279.48)	(2671.03)	(2465.3)	
R18,F10,C1	2292684	2383760	2418190	2457733.33	2451633.33	-0.25%
	(t)	(2659.54)	(2167.09)	(2779.20)	(2298.9)	
R18,F01,C2	921762	980992	964367	986701.67	966850.66	-2.01%
	(t)	(2717.04)	(2209.13)	(2810.17)	(2564.34)	
R18,F05,C2	1866215	2033500	1980170	2042536.67	2018590	-1.17%
	(t)	(3865.35)	(2810.98)	(4168.55)	(3032.4)	
R18,F10,C2	2895982	2981980	2928930	3011723.33	2972860	-1.29%
	(t)	(3828.76)	(3575.27)	(3939.71)	(3487.5)	
R18,F01,C8	1485690.8	1611800	1568050	1622236.67	1576570	-2.82%
	(t)	(15115.90)	(8151.71)	(19064.10)	(10321.23)	
R18,F05,C8	4010736.7	4379870	4553560	4515690.00	4690270	3.87%
	(t)	(33140.40)	(15068.2)	(34565.93)	(18654.4)	
R18,F10,C8	6663506.4	7562480	7639160	7762600.00	7998752.33	3.04%
	(t)	(31716.60)	(26296.6)	(32354.43)	(28642.6)	